

Additional Information

Any object, fully or partially immersed in a fluid, is buoyed up by a force equal to the weight of the fluid displaced by the object.

– Archimedes of Syracuse

Archimedes' principle can be represented by the following formula:

$$\text{apparent weight} = \text{actual weight} - \text{magnitude of buoyant force}$$

The buoyant force is an upward force that is created by the surrounding fluid acting on an object that has been submerged or partially submerged in the fluid. The magnitude of the buoyant force is equal to the weight of the fluid that has been displaced by the object.

The actual weight is a downward force which can be calculated by multiplying the mass of the object by the acceleration due to gravity ($W = mg$).

The apparent weight is equal to the actual weight of the object decreased by the buoyant force ($W' = W - F_b$).

Find the Hidden Message

The forces acting on the test tube are:

Gravitational force (F_g): a downward force equivalent to the weight of an object and measured in newtons. It is calculated by multiplying the mass (m) of the object by the acceleration due to gravity ($g = 9.81 \text{ m/s}^2$):

$$F_g = mg \quad (\text{Equation 1})$$

$$F_g \text{ (N)} = \text{mass (kg)} \times 9.81 \text{ m/s}^2$$

Buoyant force (F_b): an upward force created by the surrounding fluid acting on the submerged or partially submerged object. Its magnitude is equivalent to the weight of the displaced fluid (gas or liquid) measured in newtons. It is calculated by multiplying the density of the displaced fluid (ρ) by the acceleration due to gravity (g) and by the volume of the displaced fluid (v):

$$F_b = \rho gv \quad (\text{Equation 2})$$

$$F_b \text{ (N)} = \text{density (kg/L)} \times 9.81 \text{ m/s}^2 \times \text{volume (L)}$$

Recognizing that density is simply mass divided by volume, equation 2 can be rewritten as:

$$F_b = (m/v) gv \quad (\text{Equation 3})$$

$$F_b \text{ (N)} = (\text{mass (kg)} / \text{volume (L)}) \times 9.81 \text{ m/s}^2 \times \text{volume (L)}$$

This equation can be simplified to:

$$F_b = mg \quad (\text{Equation 4})$$

$$F_b \text{ (N)} = \text{mass (kg)} \times 9.81 \text{ m/s}^2$$

where m is the mass of the displaced fluid.

The sum of the buoyant force (F_b) and the gravitational force (F_g) is the net force (F_{NET}):

$$F_{NET} = F_g + F_b \quad (\text{Equation 5})$$

The net force (F_{NET}) indicates the total amount of force exerted on the test tube. The direction of the net force will determine the direction of acceleration. If the buoyant force (F_b) and the gravitational force (F_g) are equal and opposite, then the net force (F_{NET}) is zero, which means that the object will be in equilibrium and will not move.

The gravitational force (F_g) pulling down on the test tube is the same throughout the experiment because the mass remains constant.

When the test tube is floating on the surface of the water, the buoyant force (F_b) and the gravitational force (F_g) are equal and opposite, and the net force (F_{NET}) is zero. When the cap is screwed on and the test tube is pressing against the cap, the buoyant force (F_b) is greater than the gravitational force (F_g) and the net force (F_{NET}) is upwards (**Figure 1**). To relate to Archimedes' principle, an upward net force corresponds to a negative apparent weight, which means that the test tube wants to rise.

As the pop bottle is squeezed, the pressure on the bottle increases. Since water cannot be compressed, the pressure is transmitted unchanged through the water and compresses the air inside the test tube. The weight of water which is displaced decreases as water moves into the volume voided by the compressed air, resulting in a decrease in the buoyant force. The decrease in buoyant force causes the test tube to sink. To tie in Archimedes' principle, the test tube sinks when the actual weight (gravitational force) exceeds the buoyant force. When the pressure on the bottle is released, the air inside the test tube expands, causing the buoyant force to increase due to the greater volume of water which is displaced. Consequently, the test tube rises and its apparent weight drops.

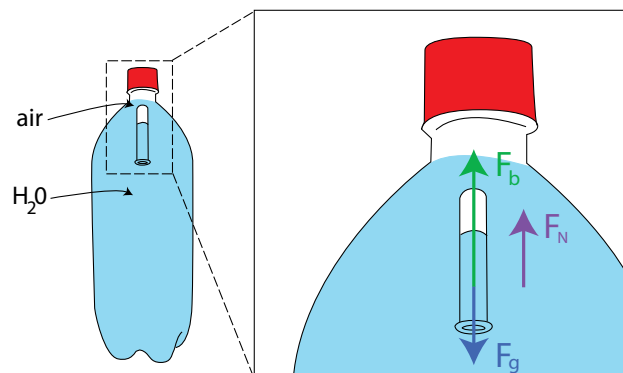


Figure 1

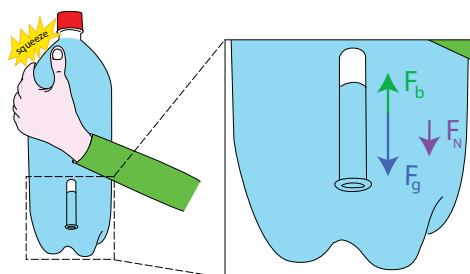


Figure 2

If the buoyant force (F_b) is a smaller value than the gravitational force (F_g), then the net force (F_{NET}) will be downwards. On the other hand, if the magnitude of the buoyant force (F_b) exceeds the gravitational force (F_g), then the net force (F_{NET}) will be upwards. When the bottle is squeezed, the buoyant force (F_b) decreases and becomes smaller than the gravitational force (F_g), causing the test tube to sink (**Figure 2**). The test tube will stop moving when the two forces cancel out and the net force (F_{NET}) is zero.

Conservation of Mass

The forces acting on the system are the gravitational force (F_g) and the buoyant force (F_b).

The sum of the buoyant force (F_b) and the gravitational force (F_g) is the net force (F_{NET}), which represents the total amount of force pushing on the scale:

$$F_{NET} = F_g + F_b \quad (\text{Equation 5})$$

The gravitational force (F_g) acting on the water bottle plus balloon remains constant throughout the experiment because mass and gravity remain constant. If the buoyant force (F_b) exceeds the gravitational force (F_g), then the net force (F_{NET}) will point upwards and the bottle and balloon will rise off the balance. However, if the buoyant force (F_b) is less than the gravitational force (F_g), then the net force (F_{NET}) will point downwards (**Figure 3**). In the absence of a buoyant force, the balance reading gives the mass of the weighed objects.

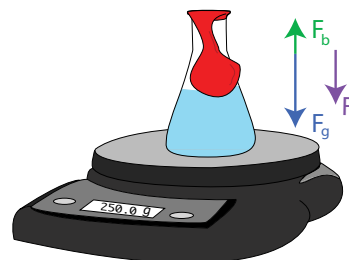


Figure 3

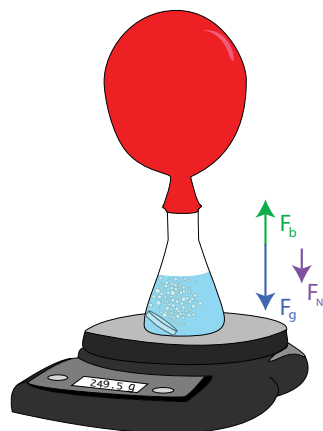


Figure 4

When antacid is added to the water in the bottle, the balloon expands, resulting in an increase in the volume of surrounding air which is displaced and consequently, an increase in the buoyant force (F_b). The net force (F_{NET}) for this situation is obtained by multiplying the reading on the balance by the acceleration due to gravity (**Figure 4**), and is decreased compared to the initial net force (F_{NET}). To relate to Archimedes' principle, the mass reading before the balloon expands is equivalent to the actual weight ($W = mg$), and the mass reading after reaction is equivalent to the apparent weight ($W' = W - F_b$).

In the second situation, the balance gives a reading which is reduced by the mass of air which has been displaced by the balloon:

$$\text{post-reaction mass} = \text{pre-reaction mass} - \text{mass of displaced fluid}$$

Therefore, the mass of displaced fluid can be determined from the difference in mass readings, before and after the reaction.

Recall that the net force can be determined by multiplying the mass readings by the acceleration due to gravity. When the initial F_{NET} (before addition of antacid) is compared to the final F_{NET}' (after addition of antacid), a noticeable difference between the magnitude (length) of the vectors ($F_{NET} > F_{NET}'$) is observed. The downward direction of the net force is however unchanged. The decrease in net force ($F_{NET} > F_{NET}'$) indicates that less force is pushing down on the balance, which means that the bottle plus balloon are not as heavy in the second situation (in other words, apparent weight is less than actual weight) although the mass remains constant. The difference in force between the two situations, $F_{NET} - F_{NET}'$ is equal to the buoyant force.

When sugar is added to the water, no reaction or displacement of surrounding air occurs. This means that F_b is zero and F_g remains constant. This means that, unlike when antacid is added, the mass of the bottle plus balloon does not change when sugar is added to the water ($m = W/g$).

Pop Floats

Regular pop in water:

Regular pop contains 35 g (~ 9 tsp) of sugar, which causes its density to be greater than the density of water. When the can of regular pop is dropped into a tank of water, the gravitational force (F_g) exceeds the buoyant force (F_b) and the can sinks to the bottom of the tank (**Figure 5**).

Diet pop in water:

Diet pop contains only 2 g (~ 1/2 tsp) of aspartame and has a density which is less than that of water. When the can of diet pop is dropped into the water, the upward buoyant force (F_b) is equal in magnitude to the downward gravitational force (F_g), resulting in a net force of zero. The forces cancel out, and the can is in equilibrium, floating statically at the surface of the water (**Figure 5**), without rising or sinking. When F_b and F_g are equal in magnitude, the apparent weight of the object is zero ($W' = F_g - F_b$).

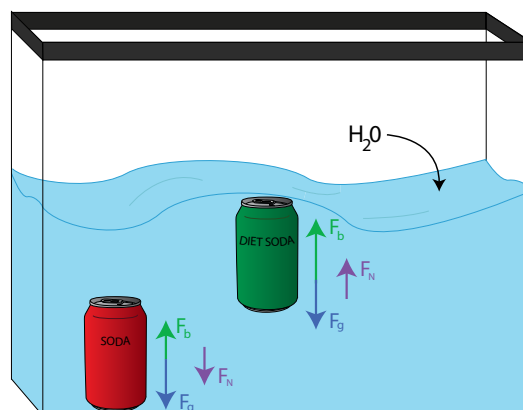


Figure 5

Regular pop in salt water:

Adding salt to the water increases the density of the fluid in the tank. As the density of the salt water increases, the buoyant force (F_b) increases, eventually causing the can to rise, until the point where the buoyant force and the gravitational force (F_g) are in equilibrium. When the density of the salt water is equal to the density of the pop, the buoyant force and the gravitational force cancel each other out ($F_{NET} = 0$) and the pop can remains suspended in the salt water, without sinking or rising. In other words, the can will rise until its actual weight ($W = mg$) and the buoyant force cancel out.

The addition of even more salt, such that the density of the salt water now surpasses the density of the pop, causes the can to rise to the surface of the water because F_b exceeds F_g . Once at the surface, the forces are again in equilibrium ($F_{NET} = 0$) and the can floats statically (**Figure 6**). In this situation, the density increase due to the addition of salt acts to increase the buoyant force and causes the can to rise; however, as the can rises, less of the surrounding fluid (salt water) is displaced and the buoyant force decreases to equilibrium. Note that if insufficient salt is added and the density of the surrounding fluid is less than the density of the pop, the can will sink.

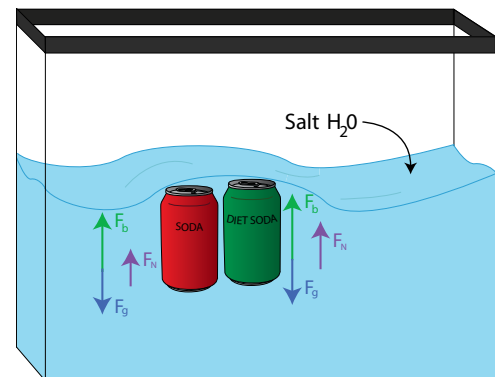


Figure 6

Diet pop in salt water:

The density of diet pop is less than that of water. As salt is added to the water in the tank, the difference between the density of the diet pop and the surrounding fluid becomes more pronounced. This increased difference in the densities causes the can of diet pop to sit higher in salt water than it does in regular water.

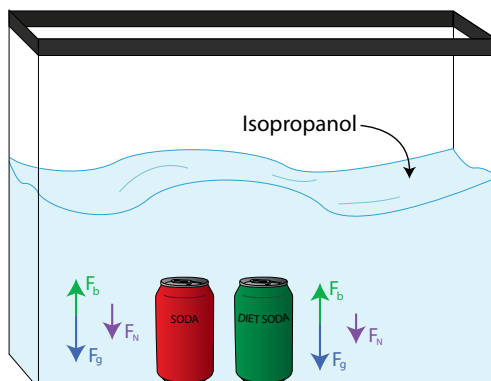


Figure 7

Regular pop and diet pop in rubbing alcohol:

In this case, the densities of both types of pop are greater than the density of rubbing alcohol. Both cans sink because the gravitational force (F_g) is larger than the buoyant force (F_b) (**Figure 7**), resulting in a downward net force.