a WOW Lab BLUEPRINT

The Human Sundial

Additional Information

Sundials

The time of year affects the way a sundial is used. The sun's path varies in height with the seasons, which is why the days are longer in summer than in winter. This can be accounted for by altering the position of the gnomon within the ellipse. The sundial changes based on the angle at which the sun's rays hit the plane of the equator. This angle, θ in **figure 1**, is needed to determine the point at which to stand inside the ellipse. The earth rotates around the sun in an elliptical orbit; this angle is constantly changing throughout the year.

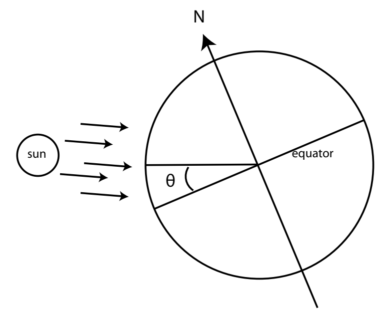

Figure 1

Ellipses

An ellipse is a closed curve that results by cutting a cone at an angle (**Figure 2**). If the centre of an ellipse is placed on the origin of the x-y plane, the equation of the line is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \text{(Equation 1)}$$

where a and b are the distances shown in **figure 3** below.

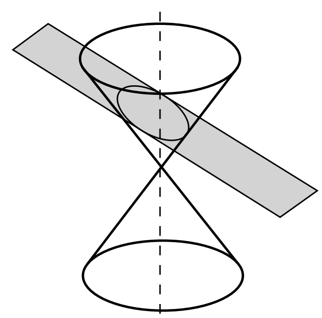

Figure 2

Introduce another variable, θ, and let x = a cos θ and y = b sin θ.

Putting these values of x and y into **equation 1** results in the following equation:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \cos^2 q + \sin^2 q = 1 \qquad \text{(Equation 2)}$$

If a and b are known, then the angle θ can be varied from 0° to 360°. By working out what x and y are for various angles, a number of data points can be plotted. These points will trace out an ellipse.

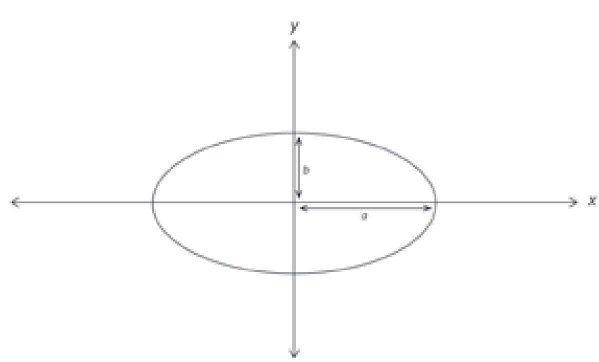

Figure 3

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How the First Method Works

Each student represents a data point. When the first student from each pair stands on the first mark of the string, the coordinates of their positions form a circle of radius b. Similarly when the second student from each pair stands on the second mark of the string, the coordinates of their positions form a circle of radius b.

As the string is kept in a straight line, the angle of both coordinates is kept constant. In other words, each pair's data points are connected by the same angle. By selecting the y-coordinate of the first point and the x-coordinate of the second point, they will be the following (from the definition of a circle): $x = a \cos \theta$ and $y = b \sin \theta$, which maps out the equation of an ellipse.

How the Second Method Works

The second method is more geometric than the first method and more accurate. To see why the method works, study **figure 6** and **figure 7**.

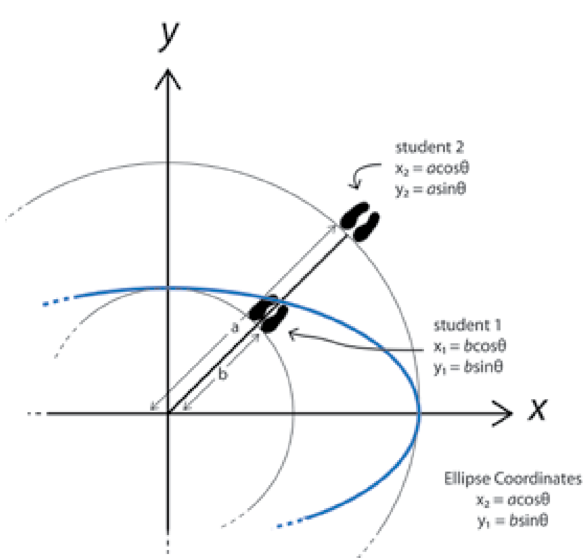

Figure 6

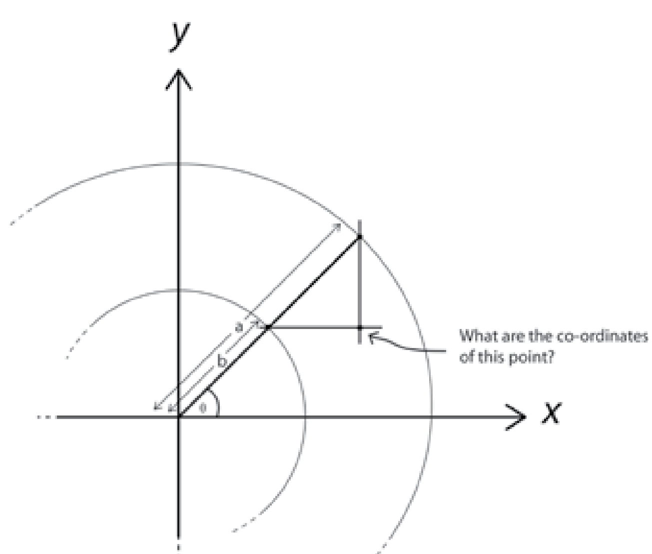

Figure 7

The Human Sundial - Additional Information

The x-coordinate, or the distance along the x-axis, can be found using the length of the string, a, and the angle θ. In **figure 8**, it is apparent that the cosine ratio, $\cos\theta = \text{opposite} \div \text{hypotenuse}$ should be used.

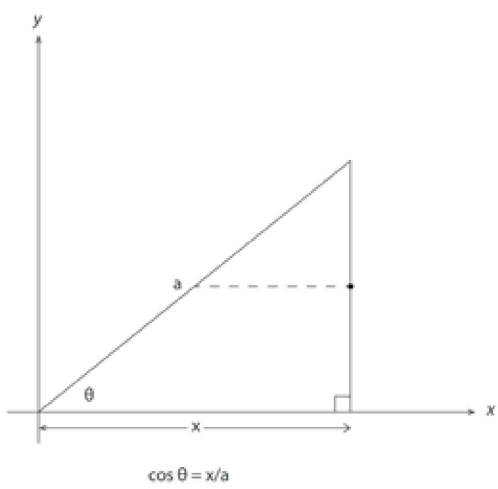

Figure 8

Similarly, the y-coordinate can be found. In **figure 9**, if the length of string b is treated as the hypotenuse of a right angled triangle, the length of the adjacent side of this triangle is the y-coordinate of the desired point. In this case, the sine ratio, $\sin\theta = \text{adjacent} \div \text{hypotenuse}$ must be used.

Putting the x and y-coordinates together, it is apparent that the point's coordinates are $x = a\cos\theta$ and $y = b\sin\theta$, exactly what was shown before mapping out an ellipse.

Note that in this second method, it was possible to control the angle of each point marked. In 24 hours, the sun completely circles the Earth (recall that there are 360º in a circle). Therefore, to find how many degrees are in 1 hour, 360º is divided by 24, giving 15º. This is why hour points are marked every 15º.

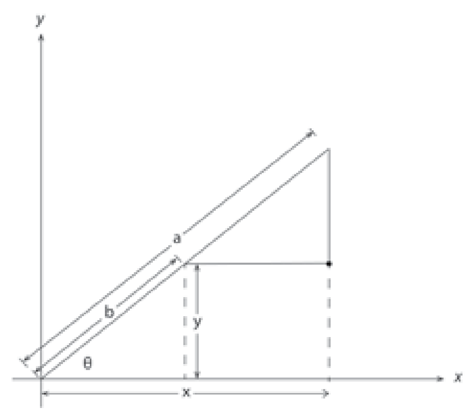

Figure 9